## INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE B.MATH - Third Year, Second Semester, 2002-03 Statistics - IV, Backpaper Examination

(10) 1. Consider an  $I \times J$  contingency table where the rows  $(1 \le i \le I)$  are independent samples. Let  $m_{ij}$  be the expected count of the (i, j) cell whose probability is  $p_{ij}$ . Prove that

$$\log(m_{ij}) = u + u_{1(i)} + u_{2(j)}, \forall i, j, \text{if and only if}$$

$$p_{1j} = p_{2j} = \dots = p_{Ij}, j = 1, 2, \dots, J.$$

(10) 2. What is the difference between

(i) fitting a normal model for a given data set, and

(ii) fitting a model for the same data set after obtaining the kernel density estimate using a Gaussian kernel?

(10) 3. Let X be  $N(\theta, 1)$ , where  $\theta < 0$ . Consider the decision problem where the loss function is  $L(\theta, a) = (\theta - a)^2$ . Consider the two decision rules,  $\delta_1(X) = X$  and  $\delta_2(X) = X^- = \min\{X, 0\}$ . Show that  $\delta_2$  has a uniformly smaller risk than  $\delta_1$  for all  $\theta < 0$ .

(10) 4. Find the minimax strategy for the two-person, zero-sum game with the following loss matrix:

	$a_1$	$a_2$	$a_3$	$a_4$
$\theta_1$	3	1	0	2
$\theta_2$	0	2.5	3	1

(10) 5. Two expert wine tasters provided the following rankings for each of the 12 different bottles of burgundy wine.

wine	1	2	3	4	5	6	7	8	9	10	11	12
Judge 1	10	8	4	11	2	5	3	12	1	9	7	6
Judge 2	7	9	2	8	3	6	10	12	4	11	5	1

Are parametric methods suitable to investigate the difference between the two rankings? What are the different approaches available for investigating this problem? Explain the procedures; there is no need to analyze the data.